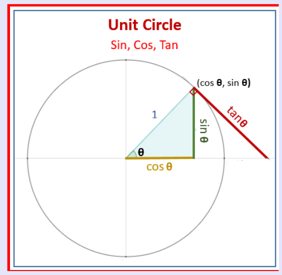
Trigonometry

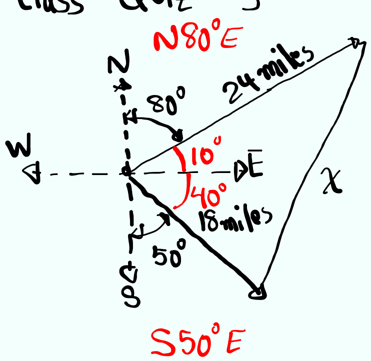
Lecture 20

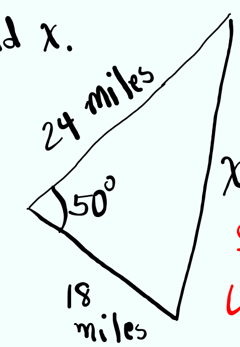


Feb 19-8:47 AM

Class Quiz 5

Find x.





SAS

Law of Cosines

$$x^2 = 24^2 + 18^2 - 2 \cdot 24 \cdot 18 \cos 50^\circ$$

$$x^2 \approx 344.632$$

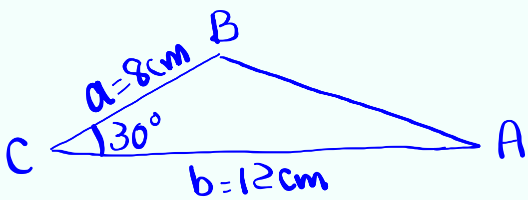
$$x \approx \sqrt{344.632} \qquad x \approx 18.564$$

$$\boxed{x \approx 19}$$

Oct 2-10:22 AM

Find the area of triangle ABC such that

$a = 8cm$, $b = 12cm$, and $C = 30°$.



SAS

Area

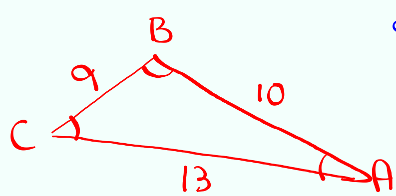
$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} \cdot 8 \cdot 12 \cdot \sin 30°$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{8}^{4} \cdot \cancel{12}^{6} \cdot \frac{1}{\cancel{2}} = \boxed{24\ cm^2}$$

Oct 2-10:42 AM

Find the area of triangle ABC such that

$a = 9in$, $b = 13in$, and $c = 10in$.



SSS

Area

Heron's Formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{P}{2} = \frac{a+b+c}{2}$

$$s = \frac{9+13+10}{2} = \frac{32}{2} = 16$$

$$\text{Area} = \sqrt{16(16-9)(16-13)(16-10)}$$

$$= \sqrt{16 \cdot 7 \cdot 3 \cdot 6} = \sqrt{2016} = 44.8998\ldots$$

$$\approx \boxed{45\ in^2}$$

Oct 2-10:46 AM

Simplify

$$\frac{\cos^3 x + \sin^2 x \cos x}{\sin x} = \frac{\cos x(\cancel{\cos^2 x + \sin^2 x})^{1}}{\sin x}$$

$$= \frac{\cos x}{\sin x} = \boxed{\cot x}$$

Simplify $\dfrac{1 + \cot A}{\csc A}$

$$= \frac{1 + \frac{\cos A}{\sin A}}{\frac{1}{\sin A}} = \frac{\sin A \cdot 1 + \cancel{\sin A} \cdot \frac{\cos A}{\cancel{\sin A}}}{\cancel{\sin A} \cdot \frac{1}{\cancel{\sin A}}}$$

LCD = Sin A

$$= \frac{\sin A + \cos A}{1} = \boxed{\sin A + \cos A}$$

Oct 2-10:54 AM

Verify $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

$$\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \sin x \cdot \underbrace{\frac{1}{\csc x}}_{\sin x} + \cos x \cdot \underbrace{\frac{1}{\sec x}}_{\cos x}$$

$$= \sin^2 x + \cos^2 x = 1$$

Oct 2-11:02 AM

Verify

$$\frac{1+\sec^2 x}{1+\tan^2 x} = 1+\cos^2 x$$

$$\frac{1+\sec^2 x}{1+\tan^2 x} = \frac{1+\frac{1}{\cos^2 x}}{1+\frac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x \cdot 1 + \cancel{\cos^2 x} \cdot \frac{1}{\cancel{\cos^2 x}}}{\cos^2 x \cdot 1 + \cancel{\cos^2 x} \cdot \frac{\sin^2 x}{\cancel{\cos^2 x}}}$$

LCD $= \cos^2 x$

$$= \frac{\cos^2 x + 1}{\cos^2 x + \sin^2 x}$$

$$= \frac{\cos^2 x + 1}{1}$$

$$= \cos^2 x + 1$$

$$= \boxed{1 + \cos^2 x}$$

Oct 2-11:06 AM

New Identities (Formula)

Addition & Subtraction

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

Oct 2-11:12 AM

Find exact Value of $\sin 75^\circ$

$75^\circ = 30^\circ + 45^\circ$

$\sin 75^\circ = \sin(30^\circ + 45^\circ) = \sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

Use $\sin(A+B)$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

Find exact Value of $\cos 15^\circ$.

$45^\circ - 30^\circ = 15^\circ$

$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

Use $\cos(A-B)$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

Oct 2-11:17 AM

Verify $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$\frac{\pi}{2} = 90^\circ$

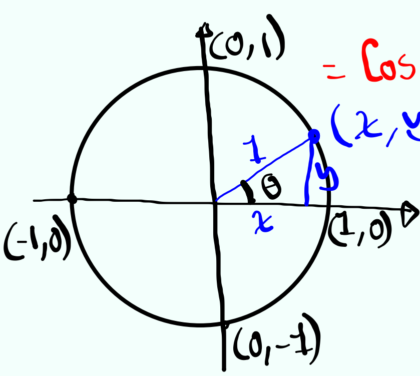
$\sin 90^\circ = 1$

$\cos 90^\circ = 0$

Use $\sin(A-B)$

$$\sin\left(\frac{\pi}{2} - x\right) = \overset{1}{\cancel{\sin\frac{\pi}{2}}}\cos x - \overset{0}{\cancel{\cos\frac{\pi}{2}}}\cdot \sin x$$

$$= \cos x - 0 = \boxed{\cos x}$$



$\sin\theta = y$ $\quad \sin 0^\circ = 0$ $\quad \sin 90^\circ = 1$

$\cos\theta = x$ $\quad \cos 0^\circ = 1$ $\quad \cos 90^\circ = 0$

Oct 2-11:26 AM

Find the Formula for tan(A+B)

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

Divid everything by Cos A Cos B

$$= \frac{\frac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} + \frac{\cancel{\cos A} \sin B}{\cancel{\cos A} \cos B}}{\cancel{\frac{\cos A \cos B}{\cos A \cos B}} - \frac{\sin A \sin B}{\cos A \cos B}}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

Oct 2-11:30 AM

Find exact Value for tan 105°

$105° = 45° + 60°$

$$\tan 105° = \tan(45° + 60°) = \frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°}$$

Use tan(A+B)

$$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

Rationalize the deno.

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{1 + \sqrt{3} + \sqrt{3} + \sqrt{9}}{1 \cancel{+\sqrt{3}} \cancel{-\sqrt{3}} - \sqrt{9}} = \frac{1 + 2\sqrt{3} + 3}{1 - 3}$$

$$= \frac{4 + 2\sqrt{3}}{-2} = \frac{\cancel{2}(2+\sqrt{3})}{-\cancel{2}} = \boxed{-(2+\sqrt{3})}$$

tan 105°

Oct 2-11:39 AM